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Impact of double-logarithmic electroweak radiative corrections on the non-singlet structure functions at small x

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Abstract: In the QCD context, the non-singlet structure functions of u and d -quarks are identical, save the initial quark densities. Electroweak radiative corrections, being flavor-dependent, bring further difference between the non-singlets. This difference is calculated in the double-logarithmic approximation and the impact of the electroweak corrections on the non-singlet intercepts is estimated numerically.

Keywords: Standard Model, Deep Inelastic Scattering, QCD.

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Contents

1. Introduction

Double-logarithmic (DL) contributions were discovered in ref. [1] in the QED context and since that have become a popular object of theoretical investigations. On one hand, DL terms are among the most sizable radiative corrections in each order of the field theories at high energies. On the other hand, the ways to select the Feynman graphs yielding DL terms, the means to calculate DL contributions and the methods of all-order summations first developed in ref. [2] converted earlier examples of DL calculations into the regular technique that allows to account for DL radiative corrections in a quite efficient and simple way. With certain technical modifications, especially non-trivial for inelastic processes, the general prescriptions of calculating DL asymptotics elaborated in ref. [2] were generalized to QCD and the Standard Model of the electro-weak interactions at TeV energies where the total energy $\sqrt{s} \gg M_{W,Z}$. As for the electro-weak (EW) double-logarithms, quite often in the literature they are accounted in fixed orders in the EW couplings.[1] Ref. [4] proved the exponentiation of the soft EW DL contributions. Such an exponentiation takes place for electro-weak reactions in the hard kinematics. The more involved Regge kinematics was studied in refs. [5, 6]. One of the most essential difference between EW and other DL calculations is the fact that the gauge symmetry of the EW interactions is partly broken and the set of the EW bosons includes the massless (photons) and massive (W,Z) particles. The DL contributions involving soft photons are infrared-divergent and are regulated with the infrared cut-off μ exactly as in QED. The value of μ is fixed in final formulas with physical considerations. DL contributions involving soft W, Z-bosons are infrared-stable and contain, instead of μ, the boson masses $M_W,\ M_Z$. The difference between M_W and M_Z can be neglected with the DL accuracy. It makes possible to use the second cut-off, M (with $M \geqslant M_W \approx M_Z$) instead of $M_W,\ M_Z$ in the DL contributions involving virtual W and Z -bosons. This approximation considerably simplifies all-order summations of EW double-logs. Another interesting topic is the interplay between the QCD and EW double-logarithmic contributions. For the $2 \to 2$ scattering in the hard kinematics it has recently been considered in ref. [7] where the impact of the first-loop EW double-logarithmic terms on the elastic $2 \to 2$ hadronic reactions ($\equiv$ EW impact) was estimated as large as 10% at energies $\sqrt{s} \sim 500$ GeV. Later, the role of sub-leading contributions was discussed in ref. [8]. In DL approach we get that the EW impact should not be neglected, however the EW impact on the elastic QCD scattering amplitudes in the first loop appears to be smaller: it is approximately 3.5% at $\sqrt{s} \lesssim 1$ TeV. On the other hand, the total resummation of the EW DL contributions to the elastic scattering $2 \to 2$ amplitudes increases the EW impact compared to the first-loop estimate: the impact comes to be about 10% at $\sqrt{s} = 1$ TeV and, growing fast with $\sqrt{s}$, it reaches 30% at $\sqrt{s} = 10$ TeV. The EW impact on the amplitudes of the inelastic $2 \to 2 + n$ -scattering of quarks can be estimated similarly. The explicit expressions for such amplitudes in QCD were obtained in ref. [9] and the generalization to the electroweak processes can be found in ref. [10].

In contrast to the exclusive processes, the interplay between EW and QCD double-logarithmic radiative corrections to the inclusive reactions has not been considered in the literature. One of interesting subjects here would be considering the EW impact on the structure functions of the Deep-Inelastic Scattering (DIS). It is clear that the EW impact on the singlet structure functions (especially on the singlet spin-independent functions $F_{1,2}$) cannot be large because the leading contributions to the singlets come from the gluon ladder graphs and gluons do not participate in the EW interactions. On the contrary, considering the EW impact on the non-singlet structure functions, where the quark ladder graphs yield main contributions, could be quite interesting. Indeed, the EW corrections depend on the flavors of the involved quarks, so accounting for these EW corrections in DLA together with the QCD background can bring qualitatively new phenomena. In order

[1] To our knowledge, one of the earliest calculations of EW double-logarithms in the first loop was done in ref. [3].

to see it, let us consider the flavor non-singlet contributions to the DIS structure functions F_1 (it describes the unpolarized DIS) and g_1 (describing the polarized DIS). Both of them are the flavor-depended contributions to the inclusive cross sections of the DIS and often addressed as the non-singlet structure functions $f^{(+)}(x, Q^2)$ and $f^{(-)}(x, Q^2)$ respectively. As is well-known, the expressions for $f^{(\pm)}(x, Q^2)$ include the initial quark densities δq, with $\delta q = \delta u,\ \delta d$, the anomalous dimensions (to describe the Q^2- evolution of the initial quark densities, converting them into the evolved quark distributions) and the coefficient functions (to describe the x -evolution of the evolved distributions). When calculated in the QCD framework, $f^{(\pm)}(x, Q^2)$ for the u- quark and d- quark coincide, save difference between $e_u^2 \delta u$ and $e_d^2 \delta d$: the quark-gluon interactions do not depend of flavors of the quarks. Electroweak corrections to $f^{(\pm)}$ bring more difference: they cause a difference in the x and Q^2 -evolutions of the initial quarks and split $f^{(\pm)}$ into $f_u^{(\pm)}$ and $f_d^{(\pm)}$ (the subscripts $u,\ d$ label the initial quark flavors). The difference in the evolutions of u and d -quarks means that $f_u^{(\pm)} \neq f_d^{(\pm)}$ even if $e_u^2 \delta u = e_d^2 \delta d$. Impact of the electromagnetic $\sim O(\alpha)$ corrections was studied in ref. [11] where DGLAP evolution equation [12] was used for accounting for the QCD corrections. However, DGLAP does not include resummation of the DL terms $\sim \alpha_s^k \ln^{2k}(1/x)$ and the single-logarithmic (SL) terms $\sim \alpha_s^k \ln^k(1/x)$. The point is that DGLAP was originally suggested for operating within the region of large x where both the double- and single- logarithms of x could easily be neglected in higher loops. Accounting for them to all orders in α_s becomes necessary in the small-x region. DGLAP lacks the resummation, so the extrapolation of DGLAP into the small-x region involves introducing the singular fits for δq with many phenomenological parameters (see e.g. ref. [13]) but suggests no theoretical explanations why δu and δd should be singular. In fact, the only role of the singular terms in the fits is to mimic the total resummation of the leading logarithms of x (see ref. [14] for more detail). When the resummation is taken into account, the singular factors should be dropped and therefore the fits can be simplified. On the other hand, the total resummation of the EW DL contributions to $f^{(\pm)}$ makes possible to estimate their impact on the small-x behavior of the non-singlets. In doing so, we follow the approach of refs. [15, 5, 6]. Through the paper we neglect the running effects for the EW couplings.

The present paper is organized as follows: in section 2 we briefly remind the results of ref. [15] for the non-singlet structure functions $f^{(\pm)}$ in QCD. The expressions for them are obtained as the solutions of the Infrared Evolution Equations (IREE). In the present paper we do not derive the IREE as this procedure can easily be found in ref. [15]. Instead, we demonstrate how the QCD-results enlisted in section 2 can be generalized to account for the EW double-logarithms. In order to do it in the simplest way, in section 3 we first extend the QCD results for $f^{(\pm)}$, adding the electromagnetic DL corrections to the QCD results of section 2. After that in section 4 we obtain the system of IREE where all electroweak DL corrections are taken into account. The evolution equations for $f_u^{(\pm)}$ and $f_d^{(\pm)}$ involve eight anomalous dimensions instead of two in QCD. They account for the total resummation of the QCD and EW- double-logarithms. We find them again with composing IREE. Those IREE are obtained and solved in section 5. Besides the anomalous dimensions, expressions for $f_{u,d}^{(\pm)}$ include coefficient functions. In order to specify them we use the matching between

$f_u^{(\pm)}$ and new amplitudes $\widetilde{f}_u^{(\pm)}$ describing the same process, however at small Q^2. They have to be calculated independently. We do it in section 6, once more with composing and solving IREE. It makes possible to obtain explicit expressions for $f_{u,d}$ first in the Mellin (momentum) space in section 7 and then in the conventional form in section 8. In section 9 we consider the small-x asymptotics of the non-singlet structure functions and estimate the impact of the EW corrections on the non-singlet intercepts. Section 10 is for concluding remarks.

2. Non-singlet structure function at small x in the QCD framework

The term "non-singlet structure functions" stands for flavor-dependent contributions to DIS structure functions. Usually, DIS structure functions are calculated with using the DGLAP evolution equations. As is known, DGLAP accounts for logarithms of Q^2 to all orders in the QCD coupling α_s and at the same time lacks the total resummation of Double- and Single logarithms (DL and SL respectively) of x. Such contributions are important at small x. The total summation of them, including the running coupling effects, was performed in refs. [15] with composing and solving the Infra-Red Evolution Equations (IREE). We will use this approach in the present paper in order to account for EW DL contributions, so we briefly remind below of the QCD results for the non-singlet structure functions. In order to make clear the fact that we discuss in this section only the QCD content of the non-singlet structure function, we will use the subscript "QCD" where it is necessary. Usually, notations (like f_{NS}) for the non-singlet structure functions bear the subscript "NS" but as through the paper we discuss the non-singlets only, we do not write the subscript "NS". We denote $f^{(+)}$ the non-singlet contribution to the unpolarized structure function F_1 and use the notation $f^{(-)}$ for the non-singlet contribution to the spin structure function g_1. As is known, the latter coincides with the structure function f_3. Technically, it is convenient to introduce the forward Compton amplitudes $T^{(\pm)}(s, Q^2)$ related to $f^{(\pm)}$ by the Optical theorem:

$$f^{(\pm)}(x, Q^2) = \frac{1}{\pi} \Im T^{(\pm)}(s, Q^2) \tag{2.1}$$

where we have used the standard notations: q is the momentum of the incoming virtual photon, p is the incoming quark momentum, $Q^2 = -q^2$, $x = Q^2/2pq$, $s = (p+q)^2 \approx 2pq$ when $x \ll 1$. The superscripts "$\pm$" in eq. (2.1) manifest that amplitudes $T^{(\pm)}$ have the signatures $\pm$. It means that they are defined as follows:

$$T^{(\pm)} = \frac{1}{2}[T(s, Q^2) \pm T(-s, Q^2)] \ . \tag{2.2}$$

Using the signature amplitudes at high energies is absolutely necessary from the point of view of the phenomenological Regge theory and at the same time it is convenient technically (see e.g. ref. [15] for detail). Accounting for the summation of the DL contributions $\sim (\alpha_s \ln^2(1/x))^k$, $(k = 1, \ldots)$ makes necessary introducing an infrared cut-off μ. For the sake of simplicity we identify it with the starting point of the Q^2-evolution, though it is not necessary. Therefore, both $T^{(\pm)}$ and $f^{(\pm)}$ depend on μ as well. It is convenient (see ref. [15] for detail) to use an integral transform to represent $f^{(\pm)}$ and $T^{(\pm)}$. The Regge pole theory suggests that it should be the Sommerfeld-Watson transform. At $s \to \infty$ one can use its asymptotic form that looks quite similarly to the Mellin transform:

$$T^{(\pm)} = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega) F^{(\pm)}(\omega, y) \tag{2.3}$$

where the signature factors

$$\xi^{(\pm)} = [e^{-\imath\pi\omega} \pm 1]/2 \approx [1 \pm 1 - \imath\pi\omega]/2 \ . \tag{2.4}$$

We have used here that due to oscillations of the factor $(s/\mu^2)^{\omega}$, the main contribution in eq. (2.3) comes from the region of small ω. As eq. (2.3) partly coincides with the standard Mellin transform, it is often addressed as the Mellin transform and we will do the same through this paper. Nevertheless, we will use the transform inverse to eq. (2.3) in its proper form:

$$F^{(\pm)}(\omega, y) = \frac{2}{\pi\omega} \int_0^{\infty} d\rho e^{-\omega\rho} \Im T^{(\pm)}(\rho, y) \tag{2.5}$$

where we have introduced two new convenient variables $\rho = \ln(s/\mu^2)$ and $y = \ln(Q^2/\mu^2)$ and also used that ω in eq. (2.5) are small. Obviously, eq. (2.5) does not coincide with the standard Mellin transform. Substituting eq. (2.3) into eq. (2.1), we express the non-singlets $f^{(\pm)}$ through $F^{(\pm)}(\omega)$:

$$f^{(\pm)} = (1/2) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{\mu^2}\right)^{\omega} \omega F^{(\pm)}(\omega, y) \ . \tag{2.6}$$

Evolving amplitudes $T^{(\pm)}$ with respect to μ allows one to compose IREE for them. It was shown in ref. [15] that in the QCD framework the forward Compton amplitudes $T^{(\pm)}$ obey the following equation:

$$T^{(\pm)} = T^{(\pm)}_{\mathrm{Born}} + M_0^{(\pm)} \otimes T^{(\pm)} \tag{2.7}$$

where $T^{(\pm)}_{\mathrm{Born}}$ is $T^{(\pm)}$ in the Born approximation, $M_0^{(\pm)}$ are amplitudes of the forward quark-quark scattering. They should be calculated independently. After differentiating eq. (2.7) with respect to μ and applying the Mellin transform, eq. (2.7) converts into the following equation in terms of $F^{(\pm)}_{\mathrm{QCD}}(\omega, y)$:

$$(\omega + \partial/\partial y) F^{(\pm)}_{\mathrm{QCD}} = [1 + \omega/2] H^{(\pm)}_{\mathrm{QCD}}(\omega) F^{(\pm)}_{\mathrm{QCD}} \ . \tag{2.8}$$

The Born term $T^{(\pm)}_{\mathrm{Born}}$ does not depend on μ and vanishes after the differentiation. The term $\omega/2$ in eq. (2.8) describes the single-logarithmic contribution. As our aim is studying DL contributions, we will neglect such SL contributions through the paper, though we will keep α_s running. $H^{(\pm)}_{\mathrm{QCD}}(\omega)$ in eq. (2.8) are related to amplitudes $M_0^{(\pm)}$ through the Mellin transform. They are new anomalous dimensions. They include the total resummation of DL and SL QCD contributions. IREE for $H^{(\pm)}_{\mathrm{QCD}}$ is obtained in ref. [15]. When the SL terms that do not contribute to α_s are neglected, the IREE for $H^{(\pm)}_{\mathrm{QCD}}$ is

$$\omega H^{(\pm)}_{\mathrm{QCD}} = \frac{b^{(\pm)}_{\mathrm{QCD}}}{8\pi^2} + \left(H^{(\pm)}_{\mathrm{QCD}}\right)^2 \tag{2.9}$$

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where

$$b^{(\pm)}_{\rm QCD} = a_{\rm QCD} + D^{(\pm)}_{\rm QCD}\,, \tag{2.10}$$

with

$$a_{\rm QCD} = 4\pi A(\omega) C_F\,, \qquad A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\right] \tag{2.11}$$

and

$$D^{(\pm)}_{\rm QCD}(\omega) = \left(-\frac{C_F}{2N}\right)(-4)\int_0^\infty d\rho e^{-\omega\rho}\,\Re[\alpha_s(s)\mp\alpha_s(-s)]\int_{\mu^2}^s \frac{dk^2_\perp}{k^2_\perp}\alpha_s(k^2_\perp)\,. \tag{2.12}$$

Performing integration over $k^2_\perp$ in eq. (2.12), we obtain the following expression for $D^{(\pm)}(\omega)_{\rm QCD}$:

$$D^{(\pm)}_{\rm QCD}(\omega) = \frac{2C_F}{b^2 N}\int_0^\infty d\rho e^{-\omega\rho}\ln\left(\frac{\rho+\eta}{\eta}\right)\left[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} \mp \frac{1}{\rho+\eta}\right]\,. \tag{2.13}$$

In eqs. (2.11), (2.13) $\rho = \ln(s/\mu^2)$, $\eta = \ln(\mu^2/\Lambda^2_{\rm QCD})$, and we have used the standard notations: $C_F = (N^2-1)/2N = 4/3$ and b is the first coefficient of the Gell-Mann-Low function.

Eqs. (2.7)–(2.13) were obtained and discussed in detail in ref. [15], so in the present paper we do not derive them. Instead, we show in next sections how to extend the QCD results, eqs. (2.7)–(2.13), to the Standard Model of electroweak interactions. Nevertheless, let us briefly comment on them. The term $a_{\rm QCD}/\omega$ in eqs. (2.8), (2.10) is the Born contribution to the amplitudes of the forward quark-quark scattering, so that $A(\omega)$ is related to α_s through the Mellin transform of eq. (2.5). The DGLAP- parametrization prescribes that $\alpha_s = \alpha_s(k^2_\perp)$. As shown in ref. [17], this parametrization should be used at large x only. At the small-x region α_s in each rung depends rather on the horizontal gluon virtuality than on $k_\perp$ of the quarks. Such virtualities are time-like, so they participate in the Mellin transform and as a consequence, $\Im\alpha_s$ and $\Re\alpha_s$ acquire the π^2 -terms appeared in eqs. (2.11)–(2.13). The Born contribution is absent in eq. (2.8) because it does not depend on μ and therefore vanishes under differentiation over μ. The second term, $D(\omega)$ in eq. (2.10) represents the approximative DL contribution of non-ladder Feynman graphs[2] when the s and u -channel gluons with small transverse momenta are factorized so that their propagators are attached to the external quark lines (see ref. [15] for detail). Such terms are absent in eq. (2.8) because gluon propagators cannot be attached to the photon lines. The last term in the both eqs. (2.8), (2.9) corresponds to the case when a t -channel intermediate quark-antiquark pair factorizes amplitude T into a convolution of two on-shell amplitudes. When α_s is kept fixed, $A(\omega)$ is replaced by α_s and $D^{(\pm)}_{\rm QCD}$ of eq. (2.13) is changed to[3]

$$\tilde{D}^{(\pm)}_{\rm QCD} = \left(-\frac{C_F}{2N}\right)\left(-\frac{4\alpha_s^2}{\omega^2}\right)[1\mp 1]\,. \tag{2.14}$$

[2]Through this paper we use the Feynman gauge.

[3]The sign of eq. (31) in ref. [15] is wrong, however this misprint does not affect the results of the paper.

The relation $\tilde{D}^{(+)}_{\rm QCD} = 0$ means that DL contributions of the non-ladder Feynman graphs cancel each other in expressions for the forward scattering amplitudes with the positive signatures. It was first noticed in ref. [18] in the QED context and remains true in QCD when α_s is fixed. According to eq. (2.13), accounting for the running α_s effects violates it. The expression (2.14) for $\tilde{D}^{(-)}_{\rm QCD}$ (as well as eq. (2.12) for $D^{(\pm)}_{\rm QCD}$) consists of two factors (each in the brackets). The first factor $(-C_F/2N)$ comes from simplifying the color structure $t_a t_b t_a t_b$ of the involved graphs ($t_{a,b}$ are the SU(3)-generators) whereas the second factor comes from integration over momenta of the virtual partons. The terms in squared brackets in eq. (2.13) correspond to $[\alpha_s(s) \pm \alpha_s(-s)]$ and the logarithm in that equation corresponds to integral of $\alpha_s(k^2_\perp)/k^2_\perp$. We ought to draw attention that the definition of $D_{\rm QCD}$ in eq. (2.13) differs from the definition of D in ref. [15]: $D_{\rm QCD} = \omega D$. Solution to eq. (2.9) is

$$H^{(\pm)}_{\rm QCD} = \frac{\omega - \sqrt{\omega^2 - B^{(\pm)}_{\rm QCD}}}{2}\,, \tag{2.15}$$

with

$$B^{(\pm)}_{\rm QCD} = 4b^{(\pm)}_{\rm QCD}/(8\pi^2) = [4\pi A C_F + D^{(\pm)}]/(2\pi^2)\ . \tag{2.16}$$

Similarly to the DGLAP equations, the general solution to eq. (2.8) predicts the Q^2-dependence of the non-singlets. In order to fix their x-dependence, the general solutions should be specified. In other words, the coefficient functions should be found. In order to do it, we use (see ref. [15]) the matching

$$F^{(\pm)}_{\rm QCD}(\omega, y)|_{y=0} = \widetilde{F}^{(\pm)}_{\rm QCD}(\omega)\,, \tag{2.17}$$

with $\widetilde{F}^{(\pm)}_{\rm QCD}$ corresponding to the DIS off a nearly on-shell photon (with $Q^2 = \mu^2$), i.e. in the kinematics where the Q^2-dependence is neglected. Obviously, $\widetilde{F}^{(\pm)}_{\rm QCD}$ coincide with the non-singlet coefficient functions in the ω-space. We calculate them again with composing new IREE (cf eq. (2.8)):

$$\omega \widetilde{F}^{(\pm)}_{\rm QCD} = e^2_q \delta q(\omega) + H^{(\pm)}_{\rm QCD} \widetilde{F}^{(\pm)}_{\rm QCD} \tag{2.18}$$

where e_q is the electric charge of the initial quark and $\delta q(\omega)$ is the initial quark density in the ω -space. In contrast to eq. (2.8), there is the Born contribution in the rhs of eq. (2.18) because in this case we keep $Q^2 \sim \mu^2$, so the Born term depends on μ and does not vanish when differentiated with respect to μ.

Eventually we arrive at the final formula for the non-singlet structure functions $f^{(\pm)}_{\rm QCD}$ in QCD:

$$f^{(\pm)}_{\rm QCD} = \frac{e^2_Q}{2} \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^\omega \frac{\omega}{\omega - H^{(\pm)}_{\rm QCD}}\, \delta q\, e^{y H^{(\pm)}_{\rm QCD}}\ . \tag{2.19}$$

Although eq. (2.19) is obtained for $Q^2 \gg \mu^2$, the shift $Q^2 \to Q^2 + \mu^2$ generalizes eq. (2.19) to the small-Q^2 region (see ref. [16] for detail). The small-x asymptotics of $f^{(\pm)}_{\rm QCD}$ is

$$f^{(\pm)}_{\rm QCD} \sim (1/x)^{\Delta^{(\pm)}_{\rm QCD}} \tag{2.20}$$

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where $\Delta^{(\pm)}_{\text{QCD}}$ are called the intercepts. Straightforwardly they can be found with applying the saddle-point method to eq. (2.19). The shorter way is to solve the equation

$$\omega^2 - B^{(\pm)}_{\text{QCD}} = 0 \tag{2.21}$$

for the leading singularity position and to choose its largest root. The root corresponds to the rightmost singularity of eq. (2.19). Ref. [15] reads that $\Delta^{(+)}_{\text{QCD}} = 0.39$ and $\Delta^{(-)}_{\text{QCD}} = 0.42$.

3. Electromagnetic DL corrections to the non-singlet structure functions

As exchanges of virtual gluons cannot be isolated from the virtual photon exchanges, it is necessary to add the electromagnetic (EM) DL contributions to the QCD expression of eq. (2.19) for the non-singlet structure functions. Generalization of eq. (2.8) for amplitudes $T^{(\pm)}$ to account for exchanges of virtual gluons and photons can be done in a very simple way: with replacing $H^{(\pm)}_{\text{QCD}}$ by new non-singlet anomalous dimensions $h^{(\pm)}_{\text{EM}}$ accounting for both EM and QCD DL contributions. The IREE for $h^{(\pm)}_{\text{EM}}$ is similar to eq. (2.9):

$$\omega h^{(\pm)}_{\text{EM}}(\omega) = \frac{b_{\text{EM}}}{8\pi^2} + (h^{(\pm)}_{\text{EM}}(\omega))^2 \ . \tag{3.1}$$

It changes eq. (2.19) for a quite similar expression

$$f^{(\pm)}_{\text{EM}} = \frac{e_q^2}{2} \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^{\omega} \frac{\omega}{\omega - H^{(\pm)}_{\text{EM}}} \, \delta q \, e^{yH^{(\pm)}_{\text{EM}}} \tag{3.2}$$

where new anomalous dimension $H^{(\pm)}_{\text{EM}}$ sums the both QCD and EM double logarithms. It also looks like $H^{(\pm)}_{\text{QCD}}$:

$$H^{\pm}_{\text{EM}} = \frac{\omega - \sqrt{\omega^2 - B^{(\pm)}_{\text{EM}}}}{2} \, , \tag{3.3}$$

Similarly to eq. (2.16), $B^{(\pm)}_{\text{EM}}$ is expressed through $b^{(\pm)}_{\text{EM}}$:

$$B^{(\pm)}_{\text{EM}} = b^{(\pm)}_{\text{EM}}/(2\pi^2) \ . \tag{3.4}$$

Now let us specify $b^{(\pm)}_{\text{EM}}$:

$$b^{(\pm)}_{\text{EM}} = b^{(\pm)}_{\text{QCD}} + a_{\gamma} + D^{(\pm)}_{\text{EM}} \tag{3.5}$$

where a_{γ} is the electric charge of the quark:

$$a_{\gamma} = e_q^2 = 4\pi\alpha Q_q^2 \tag{3.6}$$

and

$$D^{(\pm)}_{\text{EM}} = D^{(\pm)}_{g\gamma} + D^{(\pm)}_{\gamma g} + D^{(\pm)}_{\gamma\gamma} \, , \tag{3.7}$$

with

$$D^{(\pm)}_{g\gamma} = -\frac{4\alpha Q_q^2 C_F}{b}[1 \mp 1] e^{\omega\eta} \int_{-1}^{\infty} dt e^{-\omega\eta t} \ln t \, , \qquad D^{(\pm)}_{\gamma\gamma} = -\frac{4\alpha^2 Q_q^4 [1 \mp 1]}{\omega^2} \, , \tag{3.8}$$
$$D^{(\pm)}_{\gamma g} = -\frac{4\alpha Q_q^2 C_F}{b} \int_0^{\infty} d\rho e^{-\omega\rho} \left[\frac{\rho(\rho+\eta)}{(\rho+\eta)^2 + \pi^2} \mp \frac{\rho}{\rho+\eta} \right] .$$

When α_s is fixed, the expressions for $D_{g\gamma}^{(\pm)}$ and $D_{\gamma g}^{(\pm)}$ become more simple:

$$D_{g\gamma}^{(\pm)} = D_{\gamma g}^{(\pm)} = -\frac{4\alpha Q_q^2 \alpha_s C_F}{\omega^2}[1 \mp 1] \ . \tag{3.9}$$

Let us explain how $D_{ik}^{(\pm)}$ of eq. (3.8) can be obtained from the QCD expressions for $D_{\mathrm{QCD}}^{(\pm)}$ in eq. (2.12), (2.14). Eq. (2.12) reads that $D_{\mathrm{QCD}}^{(\pm)}$ contains the QCD couplings depending on different arguments.

(a) There is $\alpha(k_\perp^2)$ that comes when the soft virtual gluon with momentum $k^2 \approx -k_\perp^2$ is coupled to quarks.

(b) There is the sum $[\alpha_s(s) \mp \alpha_s(-s)]$ from the hard virtual gluon coupled to the quarks. In $D_{\gamma g}^{(\pm)}$ and $D_{g\gamma}^{(\pm)}$ one of the gluons is replaced by the photon with the same momentum. In contrast to α_s, we treat α as fixed: $\alpha = 1/137$.

Therefore, when the soft gluon is replaced by the soft photon, $\alpha(k_\perp^2)$ in eq. (2.12) should be replaced by αQ_q^2 and we arrive at $D_{\gamma g}^{(\pm)}$. Instead, when the hard gluon is replaced, $[\alpha_s(s) \mp \alpha_s(-s)]$ should be replaced by $\alpha Q_q^2 [1 \mp 1]$, the remaining integration over $k_\perp^2$ can easily be done and we obtain $D_{g\gamma}^{(\pm)}$. At last, combining both previous cases leads us to $D_{\gamma\gamma}^{(\pm)}$ where the both gluons are replaced by photons. This case is similar to eq. (2.14), save the color factor $-C_F/(2N)$. Obviously, the replacements the gluons by photons change the two-gluon color factor $t_a t_b t_a t_b = -C_F/(2N)$ for either $t_a t_a = C_F$ (for $D_{\gamma g}^{(\pm)}$ and $D_{g\gamma}^{(\pm)}$) or 1 (for $D_{\gamma\gamma}^{(\pm)}$).

In the QCD framework, the only difference between the small-x behavior of $f_u^{(\pm)}$ (for up-quarks) and $f_d^{(\pm)}$ (for down-quarks) is the difference between the initial quark densities δu and δd whereas both the x and Q^2 -evolutions of the initial up- (u) and down- (d) quark are identical, so the subscripts u and d at $f_{u,d}^{(\pm)}$ are often dropped. Accounting for EM contributions brings a difference of the both evolutions on the flavor. To mark this difference, we introduce the non-singlet structure functions, $f_u^{(\pm)}$ and $f_d^{(\pm)}$, with the subscripts showing the flavor of the initial quark. Obviously, $f_u^{(\pm)} \neq f_d^{(\pm)}$ even if $\delta u = \delta d$. As could be well-expected, eq. (3.2) shows that the impact of EM correction on the small-x behavior of $f^{(\pm)}$ is very small. Indeed, the estimate of the impact ϵ_{EM} of the EM corrections on the intercepts is:

$$\epsilon_{\mathrm{EM}}^{(+)} = \frac{\Delta_{\mathrm{EM}}^{(+)} - \Delta_{\mathrm{QCD}}^{(+)}}{\Delta_{\mathrm{QCD}}^{(+)}} \approx \epsilon_{\mathrm{EM}}^{(-)} = \frac{\Delta_{\mathrm{EM}}^{(-)} - \Delta_{\mathrm{QCD}}^{(-)}}{\Delta_{\mathrm{QCD}}^{(-)}} \approx 1\%. \tag{3.10}$$

4. Inclusion of electroweak DL contributions

In order to include into consideration all electroweak DL contributions, adding to the gluon and photon exchanges, the W and Z -exchanges, we should modify the method that we used in the previous sections by the following reasons:

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(i) As the gauge group of the electroweak interactions is broken and electroweak bosons become massless photons and massive W, Z -bosons, the non-singlet structure functions acquire dependence on the both μ and $M_{W,Z}$.

(ii) W-exchanges cause mixing of u and d -quarks, so IREE for $f_u^{(\pm)}$ and $f_d^{(\pm)}$ together with IREE for the anomalous dimensions, are not separable (as in QCD).

Before composing the IREE, let us introduce necessary notations. We use the notation g_W for the W-coupling to quarks. It does not depend on the quark flavor. On the contrary, both the photon coupling e_q and the Z -boson coupling g_{qZ} to quarks are flavor-dependent. All these coupling are expressed through the SU(3) Standard Model coupling g and the Weinberg angle θ:

$$\begin{aligned} g_{uW} &= g_{dW} \equiv g_W = g/\sqrt{2}\,, \qquad e_q = g\sin\theta_W Q_q = g\sin\theta_W (T_3 + Y/2)\,, \\ g_{qZ} &= (g/\cos\theta_W)(T_3 - Q_q \sin^2\theta_W) = (g/\cos\theta_W)(T_3\cos^2\theta_W - (Y/2)\sin^2\theta_W)\ . \end{aligned} \tag{4.1}$$

We keep through the paper the standard notations T_3, Y and Q for the isospin, hypercharge and electric charge of quarks together with the standard relation $Q = T_3 + Y/2$. We simplify the $M_{W,Z}$ -dependence of the non-singlets, assuming that in the logarithmic expressions

$$M_W \approx M_Z = M\ . \tag{4.2}$$

Again, it is convenient to introduce the Compton amplitudes $T_u^{(\pm)}$, $T_d^{(\pm)}$ related to the non-singlet structure functions by eq. (2.1). We will address them as the forward Compton amplitudes, although at energies $\sqrt{s} \gg M_{W,Z}$ and $Q^2 \gtrsim M^2_{W,Z}$ the lepton and hadron participating in the DIS can exchange with γ, Z (neutral lepton currents) and W (charged lepton currents). In order to avoid overloading the paper we consider only the case of small Q^2:

$$Q^2 \ll M^2_{W,Z} \tag{4.3}$$

where the photon exchange between the lepton and quarks prevails. The other cases can be considered quite similarly. Under the approximation of eq. (4.2), the non-singlet functions $f_{u,d}^{(\pm)}$ and the Compton amplitudes $T_{u,d}^{(\pm)}$ depend on s, Q^2 and the mass scales μ and M. We assume the following relations between the parameters s, Q^2, M^2, μ^2:

$$s \gg M^2 \gtrsim Q^2 \gg \mu^2\ . \tag{4.4}$$

It is convenient to introduce the amplitudes $F_{u,d}^{(\pm)}(\omega, y, z)$ related to amplitude $T_{u,d}^{(\pm)}$ similarly to eq. (2.6):

$$T_{u,d}^{(\pm)} = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath}\left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega) F_{u,d}^{(\pm)}(\omega, y, z) \tag{4.5}$$

where new variable z is introduced: $z = \ln(M^2/\mu^2)$. In accounting for DL contributions, μ acts as an infrared cut-off for DL terms involving soft gluons and photons whereas M acts as the second cut-off when DL terms involving soft W, Z -bosons are considered. In contrast to the considered above QCD and EM cases, IREE for $F_{u,d}^{(\pm)}(\omega, y, z)$ involve

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the matrix of new anomalous dimensions $h^{(\pm)}_{ik}$, with i,k being $= u,d$, and involve the derivatives with respect to y and z:

$$(\omega + \partial/\partial y + \partial/\partial z)F^{(\pm)}_u = h^{(\pm)}_{uu}(\omega,z)F^{(\pm)}_u + h^{(\pm)}_{ud}(\omega,z)F^{(\pm)}_d,$$
$$(\omega + \partial/\partial y + \partial/\partial z)F^{(\pm)}_d = h^{(\pm)}_{du}(\omega,z)F^{(\pm)}_u + h^{(\pm)}_{dd}(\omega,z)F^{(\pm)}_d \ . \tag{4.6}$$

The anomalous dimensions $h^{(\pm)}_{ik}$ should be calculated independently. After they have been found, it is possible to find general solutions to eqs. (4.6). In order to specify them, i.e. to find new coefficient functions, we will follow the same use the matching

$$F^{(\pm)}_{u,d}(\omega,y,z)|_{y=0} = \widetilde{F}^{(\pm)}_{u,d}(\omega,z) \tag{4.7}$$

with the amplitudes $\widetilde{F}^{(\pm)}_{u,d}(\omega,z)$. They describe the forward Compton scattering, with the EW DL corrections accounted for, in the case when the external photon has the virtuality $\sim \mu^2$, i.e. almost on-shell. $\widetilde{F}^{(\pm)}_{u,d}$ should be found independently (cf eq. (2.17)). So, before solving eqs. (4.6) we should find $h^{(\pm)}_{ik}$ and $\widetilde{F}^{(\pm)}_{u,d}$. On this step we are going to simplify our notations. Trough the paper we keep the DL accuracy. It gives us the right to neglect terms mixing amplitudes with different signatures. Therefore, all IREE we compose are separable in the signatures (see eqs. (2.8), (2.9) and eqs. (4.6), (A.1)). So, in what follows we basically drop the signature superscripts "$(\pm)$" but restore them when it is necessary.

5. Electroweak anomalous dimensions h_{ik}

Now let us focus on obtaining explicit expressions for h_{ik}. We will do it with obtaining and solving appropriate IREE. In subsection **A** we compose IREE for h_{ik}. In contrast to the QCD -case, they are partial differential equations. The general solutions to them are found in subsection **B** and are specified in subsection **C**.

5.1 IREE for the anomalous dimensions h_{ik}

In our approach, in contrast to DGLAP, the anomalous dimensions can be found with composing and solving appropriate IREE for them. Equations for h_{ik} can be obtained as a generalization of eq. (3.1):

$$\begin{aligned}
\big(\omega + \partial/\partial z\big)h_{uu} &= b^{\rm EM}_{uu}/(8\pi^2) + h^2_{uu} + h_{ud}h_{du}\,,\\
\big(\omega + \partial/\partial z\big)h_{ud} &= b^{\rm EM}_{ud}/(8\pi^2) + h_{uu}h_{ud} + h_{ud}h_{dd}\,,\\
\big(\omega + \partial/\partial z\big)h_{du} &= b^{\rm EM}_{du}/(8\pi^2) + h_{du}h_{uu} + h_{du}h_{dd}\,,\\
\big(\omega + \partial/\partial z\big)h_{dd} &= b^{\rm EM}_{dd}/(8\pi^2) + h^2_{dd} + h_{ud}h_{du}\ .
\end{aligned} \tag{5.1}$$

The electromagnetic terms $b^{\rm EM}_{uu}$ and $b^{\rm EM}_{dd}$ in eq. (5.1) are actually defined in eq. (3.5):

$$b^{\rm EM}_{uu} = b_{\rm QCD} + a^{\rm EM}_{uu} + D^{\rm EM}_{uu}\,, \qquad b^{\rm EM}_{dd} = b_{\rm QCD} + a^{\rm EM}_{dd} + D^{\rm EM}_{dd}\,, \qquad b^{\rm EM}_{ud} = b^{\rm EM}_{du} = 0 \tag{5.2}$$

where

$$a^{\rm EM}_{uu} = 4\pi\alpha Q^2_u\,, \qquad a^{\rm EM}_{dd} = 4\pi\alpha Q^2_d \tag{5.3}$$

and $D^{\rm EM}_{uu}, D^{\rm EM}_{dd}$ can similarly be taken from eqs. (3.7), (3.8), replacing Q_q by Q_u and Q_d respectively. We remind that we have dropped the signature superscripts "$\pm$" for the sake of simplicity. The fact that $b^{\rm EM}_{du} =^{\rm EM}_{ud}= 0$ simplifies the system in eq. (5.1). It is convenient to re-write eq. (5.1) in terms of symmetrized combinations $h_{S,A}$ and $b^{\rm EM}_{S,A}$ defined as follows:

$$h_S = h_{uu} + h_{dd}\,, \qquad h_A = h_{uu} - h_{dd}\,, \qquad b^{\rm EM}_S = b^{\rm EM}_{uu} + b^{\rm EM}_{dd}\,, \qquad b^{\rm EM}_A = b^{\rm EM}_{uu} - b^{\rm EM}_{dd}\,, \tag{5.4}$$

and to introduce h:

$$h = -\omega + h_S\ . \tag{5.5}$$

In these terms eq. (5.1) takes the simpler form:

$$\begin{aligned} \frac{\partial h}{\partial z} &= b^{\rm EM}_S/(8\pi^2) + \frac{1}{2}h^2 + \frac{1}{2}h_A^2 - \frac{\omega^2}{2} + 2h_{ud}h_{du}\,, \\ \frac{\partial h_A}{\partial z} &= b^{\rm EM}_A/(8\pi^2) + h_A h\,, \qquad \frac{\partial h_{ud}}{\partial z} = h_{ud}h\,, \qquad \frac{\partial h_{du}}{\partial z} = h_{ud}h\ . \end{aligned} \tag{5.6}$$

Eq. (5.6) reads that $h_{ud} = h_{du}$.

5.2 General expressions for h_{ik}

Eqs. (5.1), (5.6) for h_{ik} are non-linear,so solving them exactly is a quite serious technical problem. We do not pursue this aim in the present paper. Instead, we suggest an approximative procedure based on the obvious fact that the QCD coupling is greater than the electroweak ones. It means that in eqs. (5.1), (5.6)

$$b^{\rm EM}_S \gg b^{\rm EM}_A\,, b_{ud}\,, b_{du}\ . \tag{5.7}$$

Then, eq. (5.7) allows to conclude that

$$h_S \gg h_A\,, h_{ud}\,, h_{du}\ . \tag{5.8}$$

Using this relation, we can neglect h_A^2 and $h_{ud}h_{du}$ compared to h_S^2 in the rhs of the first of equations eqs. (5.6) and write an approximation for eqs. (5.6):

$$\begin{aligned} \frac{\partial h}{\partial z} &= \frac{b^{\rm EM}_S}{8\pi^2} - \frac{\omega^2}{2} + \frac{1}{2}h^2\,, \qquad & \frac{\partial h_A}{\partial z} &= \frac{b^{\rm EM}_A}{8\pi^2} + h_A h\,, \\ \frac{\partial h_{ud}}{\partial z} &= h_{ud}h\,, & \frac{\partial h_{du}}{\partial z} &= h_{ud}h\ . \end{aligned} \tag{5.9}$$

The first of eqs. (5.9) is the Riccatti equation and the others are linear, so they can be easily solved. The general solution for h_S can be written as

$$\begin{aligned} h_S(\omega,z) &= \omega + \lambda\frac{1 + C_S e^{\lambda z}}{1 - C_S e^{\lambda z}}\,, \qquad h_{ud} = h_{du} = C_{ud}\exp\int_0^z dt h(\omega,t)\,, \\ h_A &= \left[\frac{b^{\rm EM}_A}{8\pi^2}\int_0^z dt \exp\left(-\int_0^t dt' h(\omega,t')\right) + C_A\right]\exp\int_0^z dt h(\omega,t)\,, \end{aligned} \tag{5.10}$$

with $\lambda = \sqrt{\omega^2 - 2b^{\rm EM}_S/(8\pi^2)}$. $C_S, C_A(\omega)$ and $C_{ud}(\omega)$ being an arbitrary functions of ω. They have to be specified. We do it, invoking the matching

$$h_{ik}(\omega,z)|_{z=0} = H_{ik}(\omega) \tag{5.11}$$

where $H_{ik}(\omega)$ are the auxiliary anomalous dimensions corresponding to the unbroken electroweak symmetry where that W, Z -bosons are massless, so the cut-off μ is applied to all virtual bosons. These anomalous dimensions account for the total resummation of EW and QCD double-logarithms and have to be calculated independently. Using the matching of (5.11) for $h_{ik}(\omega, z)$ in eqs. (5.10), we express the unknown functions C_S, C_A, C_{ud} in terms of H_{ik}:

$$C_S = -(\lambda - H)/(\lambda + H)\,, \qquad C_A = H_A\,, \qquad C_{ud} = H_{ud} \tag{5.12}$$

where similarly to eqs. (5.4), (5.5) we have denoted

$$H = -\omega + H_S, \quad H_S = H_{uu} + H_{dd}, \quad H_A = H_{uu} - H_{dd}\ . \tag{5.13}$$

Explicit expressions for $H_{ik}(\omega)$ are obtained in appendix A.

5.3 Specifying general expressions for h_{ik}

Combining eq. (A.1) with eq. (5.12) and substituting them into eq. (5.10) leads to explicit expressions for h_{ik}:

$$\begin{aligned} h_S(\omega, z) &= \omega + \lambda \frac{(\lambda - E) - (\lambda + E)e^{\lambda z}}{(\lambda - E) + (\lambda + E)e^{\lambda z}}\,, \qquad h_{ud} = h_{du} = \frac{\widetilde{b}_{ud}}{E} \exp \int_0^z dt h(\omega, t)\,, \\ h_A &= \frac{b_A^{\mathrm{EM}}}{8\pi^2} \int_0^z dt \exp\Big(- \int_0^t dt' h(\omega, t')\Big) \exp \int_0^z dt h(\omega, t)\ . \end{aligned} \tag{5.14}$$

Denoting

$$\lambda/E = \tanh\beta\,, \tag{5.15}$$

we obtain that

$$h = -\frac{\lambda}{\tanh(\lambda z/2 + \beta)}\ . \tag{5.16}$$

Substituting it into eq. (5.14) leads to explicit expressions for h_S, h_A, h_{ud}:

$$\begin{aligned} h_S &= \omega - \frac{\lambda}{\tanh(\lambda z/2 + \beta)}, \qquad h_{ud} = h_{du} = \frac{\widetilde{b}_{ud}}{E} \frac{\sinh^2\beta}{\sinh^2\big(\lambda z/2 + \beta\big)}\,, \\ h_A &= \frac{b_A^{\mathrm{EM}}}{8\pi^2} \frac{1}{2\lambda \sinh^2\big(\lambda z/2 + \beta\big)} \Big[- \lambda z - \sinh 2\beta + \sinh(\lambda z + 2\beta)\Big]\ . \end{aligned} \tag{5.17}$$

Eq. (5.17) manifests that the breaking the SU(3)$\otimes$U(1) symmetry of the electroweak gauge group leads to the non-zero h_A in contrast to the expressions eq. (A.7) obtained under the assumption of the unbroken EW symmetry.

6. Coefficient functions of the non-singlets

In the present section we calculate the coefficient functions of the non-singlets. Again we follow the pattern we used in the QCD framework: the coefficient functions are given by the amplitudes $\widetilde{F}_u$, $\widetilde{F}_d$ describing the same process at $Q^2 = \mu^2$. IREE for them are similar to eq. (4.6), save two points: the first is the absence of the y -dependence because $Q^2 = \mu^2$ for $\widetilde{F}_{u,d}$ and the second is appearing initial contributions because they depend on μ at $y = 0$:

$$(\omega + \partial/\partial z)\widetilde{F}_u = e_u^2 \delta u + h_{uu}(\omega, z)\widetilde{F}_u + h_{ud}(\omega, z)\widetilde{F}_d,$$
$$(\omega + \partial/\partial z)\widetilde{F}_d = e_d^2 \delta d + h_{du}(\omega, z)\widetilde{F}_u + h_{dd}(\omega, z)\widetilde{F}_d \ . \tag{6.1}$$

The factors $\delta u, \delta d$ in eq. (6.1) stand for the initial quark densities in the ω -space. As the anomalous dimensions h_{ik} have been found in the previous section (see eq. (5.17)), we can solve eq. (6.1). Our strategy is to find a general solution to eq. (6.1) and after that to specify it with using the matching to the other auxiliary amplitudes $\phi_{u,d}$ of the same process, however obtained under the assumption of the unbroken SU(2) $\otimes$ U(1) symmetry:

$$\widetilde{F}_u|_{z=0} = \phi_u, \quad \widetilde{F}_d|_{z=0} = \phi_d \ . \tag{6.2}$$

In subsection **A** we obtain the general solution to eq. (6.1) in terms of the auxiliary amplitudes ϕ_A, ϕ_S. We calculate them in subsection **B**. It allows us to obtain the explicit expressions for amplitudes $\widetilde{F}_{u,d}$ in subsection **C**.

6.1 General solution to eq. (6.1)

Introducing the symmetrized combinations

$$\widetilde{F}_S = \widetilde{F}_u + \widetilde{F}_d, \quad \widetilde{F}_A = \widetilde{F}_u - \widetilde{F}_d, \tag{6.3}$$

we can rewrite eq. (6.1) in the symmetrical form:

$$\partial \widetilde{F}_S/\partial z = (e_u^2 \delta u + e_d^2 \delta d) + \Big(-\omega + \frac{1}{2} h_S(\omega, z)\Big)\widetilde{F}_S + h_{ud}(\omega, z)\widetilde{F}_S + \frac{1}{2} h_A(\omega, z)\widetilde{F}_A, \tag{6.4}$$
$$\partial \widetilde{F}_A/\partial z = (e_u^2 \delta u - e_d^2 \delta d) + \Big(-\omega + \frac{1}{2} h_S(\omega, z)\Big)\widetilde{F}_A - h_{ud}(\omega, z)\widetilde{F}_A + \frac{1}{2} h_A(\omega, z)\widetilde{F}_S.$$

It is easy to write down a general solution to eq. (6.4) in terms of integrals of h_{ik}. However, the expressions for h_{ik} are rather complicated, which makes scarcely possible performing those integrations. Instead, we obtain an approximative solution to eq. (6.4), having noticed that according to eq. (5.17) $h_S \gg h_A, h_{ud}$. It gives us the right to drop the term $h_A \widetilde{F}_A$ in the first of eq. (6.4). After that we arrive at the following results:

$$\widetilde{F}_S = \Big[\phi_S(\omega) + c_S(\omega)\int_0^z dt e^{-\Psi(\omega,t)}\Big] e^{\Psi(\omega,z)}\,, \tag{6.5}$$
$$\widetilde{F}_A = \Big[\phi_A(\omega) + c_A(\omega)\int_0^z dt e^{-\Psi(\omega,t)} + \frac{\phi_S}{2}\int_0^z dt h_A(\omega,t) + \frac{c_S}{2}\int_0^z dt h_A(\omega,t)\int_0^t dv e^{-\Psi(\omega,v)}\Big] e^{\Psi(\omega,z)}$$

where $c_S = e_u^2 \delta u + e_d^2\, \delta d$, $c_A = e_u^2 \delta u - e_d^2\, \delta d$ and

$$\Psi(\omega, z) = \int_0^z dt \Big[-\omega + \frac{1}{2} h_S(\omega, t)\Big] = -\frac{\omega z}{2} - \ln\left(\frac{\sinh(\lambda z/2 + \beta)}{\sinh \beta}\right) . \tag{6.6}$$

Obviously, $\widetilde{F}_S = \phi_S$ and $\widetilde{F}_A = \phi_A$ at $z = 0$ in accordance with the matching of eq. (6.2). Now we should find $\phi_{S,A}$ in order to specify eq. (6.5).

6.2 Amplitudes $\phi_{u,d}$

Amplitudes $\phi_{u,d}$ describe the forward Compton scattering off u and d -quarks under the assumption of unbroken EW symmetry and with the photon being on-shell. Obviously, they obey the following IREE:

$$\begin{aligned} \omega\phi_u &= e_u^2\delta u + H_{uu}\phi_u + H_{ud}\phi_d \,, \\ \omega\phi_d &= e_d^2\,\delta d + H_{du}\phi_u + H_{dd}\phi_d \,, \end{aligned} \tag{6.7}$$

with the obvious solution:

$$\phi_S \equiv \phi_u + \phi_d = \frac{c_S}{\omega - H_{uu} - H_{ud}}\,, \qquad \phi_A \equiv \phi_u - \phi_d = \frac{c_A}{\omega - H_{uu} + H_{ud}}\,. \tag{6.8}$$

We have used in eq. (6.8) that $H_{uu} = H_{dd}$.

6.3 Specifying the general solutions for $\widetilde{F}_{S,A}$

When ϕ_A and ϕ_S are known, the general expressions in eq. (6.5) can be specified:

$$\begin{aligned} \widetilde{F}_S &= c_S\left[\frac{e^{-\omega z/2}\sinh\beta}{(\omega - H_{uu} - H_{ud})\sinh(\lambda z/2+\beta)} + \frac{4\sinh(\lambda z/4)\cosh(\lambda z/4+\beta-\varphi)}{\sqrt{\omega^2-\lambda^2}\sinh(\lambda z/2+\beta)}\right], \\ \widetilde{F}_A &= c_A\left[\frac{e^{-\omega z/2}\sinh\beta}{(\omega - H_{uu} + H_{ud})\sinh(\lambda z/2+\beta)} + \frac{4\sinh(\lambda z/4)\cosh(\lambda z/4+\beta-\varphi)}{\sqrt{\omega^2-\lambda^2}\sinh(\lambda z/2+\beta)}\right] \\ &\quad + \frac{c_S}{2}\frac{e^{-\omega z/2}}{\sinh(\lambda z/2+\beta)}\cdot\left[\frac{\sinh\beta}{(\omega - H_{uu} - H_{ud})}\int_0^z dt\, h_A(\omega,t)\right. \\ &\qquad \left. + \frac{4}{\sqrt{\omega^2-\lambda^2}}\int_0^z dt\, h_A(\omega,t)\, e^{\omega t/2}\sinh(\lambda t/4)\cosh(\lambda t/4+\beta-\varphi)\right], \end{aligned} \tag{6.9}$$

where we have used the notation

$$\lambda/\omega = \tanh\varphi\,. \tag{6.10}$$

7. Explicit expressions for the electroweak amplitudes $F_{u,d}$

In the previous sections we obtained explicit expressions for the electroweak anomalous dimensions h_{ik} and the auxiliary amplitudes $\widetilde{F}_{u,d}$. Therefore, we can now find solutions to eq. (4.6) for amplitudes $F_{u,d}$. As eq. (4.6) is quite similar to eq. (6.1), solving it can be done in the same way. Again it is convenient to introduce the symmetrized notations

$$F_S = F_u + F_d\,, \qquad F_A = F_u - F_d \tag{7.1}$$

and express the solution in terms of them. Obviously,

$$
\begin{aligned}
F_S(\omega,z-y,z)=&\widetilde{F}_S(\omega,z-y)e^{\Psi(\omega,z)-\Psi(\omega,(z-y))} \qquad (7.2)\\
=&\,c_S(\omega)\frac{e^{-\omega z/2}}{\sinh(\lambda z/2+\beta)}\left[\frac{\sinh\beta}{\omega-H_{uu}-H_{ud}}+\int_0^{z-y}dte^{\omega t/2}\sinh(\lambda t/2+\beta)\right],\\
F_A(\omega,z-y,z)=&\left[\widetilde{F}_A(\omega,z-y)+\frac{1}{2}\int_{z-y}^{z}dth_A(\omega,t)F_S(\omega,z-y,t)e^{-\Psi(\omega,t)+\Psi(\omega,z-y)}\right]e^{\Psi(\omega,z)-\Psi(\omega,z-y)}\\
=&\frac{e^{-\omega z/2}}{\sinh(\lambda z/2+\beta)}\left[c_A\left(\frac{\sinh\beta}{\omega-H_{uu}+H_{ud}}+\int_0^{z-y}dte^{\omega t/2}\sinh(\lambda t/2+\beta)\right)\right.\\
&+\frac{c_S}{2}\left(\frac{\sinh\beta}{\omega-H_{uu}-H_{ud}}\int_0^z dth_A(t)+\int_0^{z-y}dth_A(t)\int_0^t due^{\omega u/2}\sinh(\lambda u/2+\beta)\right.\\
&\left.\left.+\int_{z-y}^{z}dth_A(t)\int_0^{z-y}due^{\omega u/2}\sinh(\lambda u/2+\beta)\right)\right]
\end{aligned}
$$

where $\widetilde{F}_S$ and $\widetilde{F}_A$ are defined in eq. (6.9) and h_A is given by eq. (5.17). We remind that $c_S=e_u^2\delta u+e_d^2\,\delta d$ and $c_A=e_u^2\delta u-e_d^2\,\delta d$.

8. Expressions for the non-singlet structure functions

Now we can write down explicit expressions for the non-singlet structure functions including the total resummation of QCD and EW double-logarithmic contributions. We express the non-singlet structure function f_u of u -quark and the non-singlet structure function f_d of d -quark in terms of their symmetrized combinations f_S and f_A:

$$
f_S=f_u+f_d\,,\qquad f_A=f_u-f_d\;. \tag{8.1}
$$

Combining eqs. (2.6) and (7.2) leads us to the following expressions:

$$
\begin{aligned}
f_S&=\frac{1}{2}\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}\left(\frac{s}{\mu^2}\right)^{\omega}F_S(\omega,z,y)\,,\\
f_A&=\frac{1}{2}\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}\left(\frac{s}{\mu^2}\right)^{\omega}F_A(\omega,z,y)\;.
\end{aligned} \tag{8.2}
$$

The Mellin amplitudes $F_{S,A}$ in eq. (8.2) are given by eq. (7.2). When the non-singlet structure functions f_u,f_d are calculated in the QCD framework, the difference between them, $f_A\neq 0$, only if $c_A=e_u^2\delta u-e_d^2\,\delta d\neq 0$. Including the EW corrections changes the situation cardinally. Indeed, eq. (7.2) manifests that the expression for f_A includes the contribution proportional to c_A and, in addition, the contribution proportional to $c_S=e_u^2\delta u+e_d^2\,\delta d$. The latter contribution arises because of mixing u and d -quarks through W -boson exchanges. It means that, with the EW corrections accounted for, $f_u\neq f_d$ even if $c_A=0$. We remind that eqs. (8.2) describe f_u and f_d in the region (4.4).

9. Impact of the EW double-logarithms on the non-singlet intercepts

Let us consider the small-x asymptotics of $f_u^{(\pm)}$ and $f_d^{(\pm)}$. When they are calculated in the QCD framework, they are identical, save the difference between $e_u^2\delta u$ and $e_d^2\delta d$, and given by eq. (2.20). Accounting for the EW DL contributions keeps the Regge form of the asymptotics but changes the QCD intercepts $\Delta^{(\pm)}_{\rm QCD}$ for the new ones which we denote $\Delta^{(\pm)}$. According to eq. (2.21), the intercepts are the rightmost singularities of $F_{S,A}$ in eq. (8.2) The leading singularity is the square root branching point in eq. (A.8):

$$(\omega^2 - 4b_{uu}/(8\pi^2))^2 - 16(b_{ud}/(8\pi^2))^2 = 0\ . \tag{9.1}$$

The terms $b_{uu},\ b_{ud}$ in eq. (9.1) are defined in eq. (A.2). They depend on the signatures, so from now on we should once more write explicitly the signature superscripts " $\pm$ ". It is interesting to note that eq. (9.1) corresponds to the unbroken $\mathrm{SU(3)}\otimes\mathrm{SU(2)}\otimes\mathrm{U(1)}$ gauge symmetry and therefore can be rewritten in the following way:

$$\omega^2 = \frac{2}{\pi}\Big[A(\omega)C_F + \alpha_{\rm SU(2)}C'_F + \alpha_{\rm U(1)}(Y/2)^2\Big] + \frac{D^{(\pm)}}{2\pi^2} \tag{9.2}$$

where $\alpha_{\rm SU(2)} = \alpha/\sin^2\theta_W,\ \alpha_{\rm U(1)} = \alpha/\cos^2\theta_W$; then, $C'_F = 3/4,\ N' = 2,\ Y = 1/3$ and

$$\begin{aligned} D^{(\pm)} &= D^{(\pm)}_{\rm QCD} + \zeta\frac{2\alpha^2_{\rm SU(2)}C'_F}{\omega^2 N'} - z\frac{4\alpha^2_{\rm U(1)}Y^4}{16\omega^2} \\ &\quad -\frac{4\alpha_{\rm SU(2)}C_F C'_F}{b}\left[\int_0^\infty d\rho e^{-\omega\rho}\left(\frac{\rho(\rho+\eta)}{(\rho+\eta)^2+\pi^2}\mp\frac{\rho}{\rho+\eta}\right) + \zeta e^{\omega\eta}\int_{-1}^\infty dt e^{-\omega\eta t}\ln t\right] \\ &\quad -\frac{4\alpha_{\rm U(1)}C_F Y^2}{4b}\left[\int_0^\infty d\rho e^{-\omega\rho}\left(\frac{\rho(\rho+\eta)}{(\rho+\eta)^2+\pi^2}\mp\frac{\rho}{\rho+\eta}\right) + \zeta e^{\omega\eta}\int_{-1}^\infty dt e^{-\omega\eta t}\ln t\right] \\ &\quad -\zeta\frac{8\alpha_{\rm SU(2)}\alpha_{\rm U(1)}C'_F Y^2}{4\omega^2}\ . \end{aligned} \tag{9.3}$$

In eq. (9.3) we have denoted $\zeta = [1\mp 1]$.

When α_s is assumed fixed, eq. (9.2) looks more simple:

$$\omega^2 - a - d^{(\pm)}/\omega^2 = 0\,, \tag{9.4}$$

with

$$\begin{aligned} a &= \frac{8\alpha_s}{3\pi} + \frac{3\alpha}{2\pi\sin^2\theta_W} + \frac{\alpha}{18\pi\cos^2\theta_W}\,, \qquad d^{(+)} = 0\,, \\ d^{(-)} &= \frac{1}{2\pi^2}\left[\frac{8}{9}\alpha_s^2 - 8\frac{\alpha_s\alpha}{\sin^2\theta_W} - \frac{8}{27}\frac{\alpha_s\alpha}{\cos^2\theta_W} + \frac{3}{4}\frac{\alpha^2}{\sin^4\theta_W} - \frac{1}{6}\frac{\alpha^2}{\sin^2\theta_W\cos^2\theta_W} - \frac{1}{324}\frac{\alpha^2}{\cos^4\theta_W}\right]. \end{aligned} \tag{9.5}$$

Eq. (9.4) can easily be solved analytically, the solutions, $\omega_0^{(\pm)}$ are

$$\omega_0^{(+)} = \sqrt{a}\,, \qquad \omega_0^{(-)} = \sqrt{(a+\sqrt{a^2+4d^{(-)}})/2}\ . \tag{9.6}$$

On the contrary, eq. (9.2) cannot be solved analytically. Numerical solutions to eq. (9.2) depend on η and their maximums which we call the intercepts[4] are

$$\Delta^{(+)} = 0.373\,, \qquad \Delta^{(-)} = 0.354\,, \tag{9.7}$$

[4]See ref. [15, 16] for detail.

while the QCD intercepts $\Delta^{(\pm)}_{\rm QCD}$ obtained in ref. [15] are

$$\Delta^{(+)}_{\rm QCD} = 0.385\,, \qquad \Delta^{(-)}_{\rm QCD} = 0.423\ . \tag{9.8}$$

However, the QCD intercepts of eq. (9.8) include both DL and single-logarithmic (SL) contributions. When, in addition to DL terms, only the SL terms contributing to α_s are taken into account and other SL terms are neglected, the QCD non-singlet intercepts $\widetilde{\Delta}^{(\pm)}_{\rm QCD}$ differ from $\Delta^{(\pm)}_{\rm QCD}$:

$$\widetilde{\Delta}^{(+)}_{\rm QCD} = 0.346\,, \qquad \widetilde{\Delta}^{(-)}_{\rm QCD} = 0.389\ . \tag{9.9}$$

Therefore, the impacts $\epsilon^{(\pm)}_{\rm QCD}$ of the SL QCD corrections on the non-singlet intercepts are

$$\epsilon^{(+)}_{\rm QCD} = \frac{\Delta^{(+)}_{\rm QCD} - \widetilde{\Delta}^{(+)}_{\rm QCD}}{\widetilde{\Delta}^{(+)}_{\rm QCD}} \approx 11\%\,, \qquad \epsilon^{(-)}_{\rm QCD} = \frac{\Delta^{(-)}_{\rm QCD} - \widetilde{\Delta}^{(-)}_{\rm QCD}}{\widetilde{\Delta}^{(-)}_{\rm QCD}} \approx 9\%\ . \tag{9.10}$$

On the other hand, impacts $\epsilon^{(\pm)}$ of the DL EW corrections on the DL QCD intercepts $\widetilde{\Delta}^{(\pm)}_{\rm QCD}$ are of the same size:

$$\epsilon^{(+)} = \frac{\Delta^{(+)} - \widetilde{\Delta}^{(+)}_{\rm QCD}}{\widetilde{\Delta}^{(+)}_{\rm QCD}} \approx 8\%\,, \qquad \epsilon^{(-)} = \frac{\Delta^{(-)} - \widetilde{\Delta}^{(-)}_{\rm QCD}}{\widetilde{\Delta}^{(-)}_{\rm QCD}} \approx -9\%\ . \tag{9.11}$$

Confronting eq. (3.10) to eq. (9.11) manifests that the impact of all EW DL corrections on the non-singlet intercepts is much greater than the impact of the electromagnetic DL corrections. It also interesting that EW DL corrections work opposite ways: they increase $\Delta^{(+)}_{\rm QCD}$ and decrease $\Delta^{(-)}_{\rm QCD}$, which makes smaller the difference between the asymptotics of the non-singlets F_1 and g_1. A qualitative explanation to that can be easily found from considering eq. (9.6): the expression for a in eq. (9.5) manifests that adding the EW terms (all they are positive) to the QCD term $8\alpha_s/3\pi$ increases a and therefore increases $\omega_0^{(+)}$ compared to its QCD value $\sqrt{8\alpha_s/3\pi}$. In contrast, there is an interplay between the increase of a and decrease of $d^{(-)}$ in the expression for $\omega_0^{(-)}$. Indeed, the QCD term $8\alpha_s^2/9$ in the expression for $d^{(-)}$ is suppressed by the negative EW contributions (the largest of them, the second term, is $\approx -40\alpha\alpha_s$). It means that $\sqrt{a^2 + 4d^{(-)}} < a$ and therefore $\omega_0^{(-)} < \omega_0^{(+)}$.

10. Conclusion

We have considered the interplay between the QCD and EW radiative corrections to the non-singlet structure functions $f^{(\pm)}$ in the double-logarithmic approximation. We accounted for the running QCD coupling effects but kept the electroweak couplings fixed. Accounting for the running EW couplings effects can be done easily. We have shown that the EW DL corrections can lead to qualitatively new phenomena which are absent in the QCD context. We have considered the EW impact on the non-singlet structure functions $f^{(\pm)}$ at small x where accounting for DL contributions is known to be absolutely necessary.

In order to calculate $f^{(\pm)}$ taking into account both QCD and EW corrections in the DLA, we applied the same method of composing Infrared Evolution Equations that we had used for calculating $f^{(\pm)}$ in QCD. The EW couplings to quarks are sensitive to the quark flavors, so the Q^2 and x -evolutions of u and d -quarks are different. Besides, exchanges with virtual W-bosons mix u and d -quarks. So, accounting for the EW corrections changes the QCD evolution equation of eq. (2.19) for the system of more involved equations in eq. (4.6). Instead of two non-singlet anomalous dimensions $H^{(\pm)}_{\rm QCD}$ in eq. (2.15), eq. (4.6) involves eight of them: $H^{\pm}_{ik}$, with $i, k = u, d$. They obey the system of non-linear differential evolution equations obtained in eq. (5.1). The approximative solutions to eq. (5.1) were obtained in eqs. (5.17). They were used to obtain the explicit expressions of eq. (8.1) for the non-singlet structure functions f_u and f_d in the kinematic region eq. (4.4). Besides, the expressions for $H^{\pm}_{ik}$ in eq. (8.1) can also be used to obtain amplitudes $M^{\pm}_{ik}$ of the forward annihilation of quark-antiquark pairs with flavor i into the quark-antiquark pairs with flavor k: $M^{\pm}_{ik} = 8\pi^2 H^{\pm}_{ik}$.

In the QCD context, the only difference between the non-singlet structure functions f_u and f_d is reduced to the difference in their initial densities $e^2_u \delta u$ and $e^2_d \delta d$, whereas their coefficient functions and anomalous dimensions are identical. In contrast, eqs. (8.1), (7.2) manifest that with the EW corrections taken into account, $f_u - f_d \neq 0$ even if $e^2_u \delta u = e^2_d \delta d$. Eqs. (8.1), (7.2) can also be used for estimating the x and Q^2 -dependence of the asymmetry

$$A_{ud}(x, Q^2) = \frac{f_u(x, Q^2) - f_d(x, Q^2)}{f_u(x, Q^2) + f_d(x, Q^2)} \tag{10.1}$$

in the kinematic region eq. (4.4). The small-x asymptotics f_u and f_d are of the Regge type. They have identical intercepts but different coefficients. Their intercepts are presented in eq. (9.7). It demonstrates that the EW corrections change the values of the QCD intercepts obtained in ref. [15] and reproduced in eq. (9.8). It is also interesting to notice that DL contributions of non-ladder Feyman graphs produce opposite influence on the values of the non-singlet intercepts: In the QCD framework, the intercept $\Delta^{(+)}_{\rm QCD}$ of the non-singlet contribution to the structure functions $F_{1,2}$ is less than the intercept $\Delta^{(-)}_{\rm QCD}$ of the non-singlet contribution to g_1. Eq. (9.7) shows that accounting for the EW corrections reverses this situation. Then, eqs. (9.7)–(9.9) manifest that the impact of DL EW corrections on the non-singlet intercepts is comparable with the impact of the sub-leading, i.e. single-logarithmic QCD contributions and reaches $\approx 11\%$. As the intercept is the exponent in the expressions $\sim s^{\Delta}$ for the Regge asymptotics, the 11% change of the intercept due to the EW contributions is quite substantial. Finally, we would like to stress that similar incorporating EW corrections into the QCD expressions for the flavor singlet structure functions at small x should bring really small impact because the small-x behavior of the singlets is mostly controlled by gluon contributions.

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A. Anomalous dimensions at the unbroken EW gauge symmetry

Contrary to the differential equations eqs. (5.1), the IREE for H_{ik} are algebraic because H_{ik} do not depend on z:

$$\omega H_{uu}^{(\pm)} = b_{uu}^{(\pm)}/(8\pi^2) + (H_{uu}^{(\pm)})^2 + H_{ud}^{(\pm)} H_{du}^{(\pm)}\,, \qquad \omega H_{ud}^{(\pm)} = b_{ud}^{(\pm)}/(8\pi^2) + H_{uu}^{(\pm)} H_{ud}^{(\pm)} + H_{ud}^{(\pm)} H_{dd}^{(\pm)}\,,$$
$$\omega H_{du}^{(\pm)} = b_{du}^{(\pm)}/(8\pi^2) + H_{du}^{(\pm)} H_{uu}^{(\pm)} + H_{du}^{(\pm)} H_{dd}^{(\pm)}\,, \quad \omega H_{dd}^{(\pm)} = b_{dd}^{(\pm)}/(8\pi^2) + (H_{dd}^{(\pm)})^2 + H_{ud}^{(\pm)} H_{du}^{(\pm)} \tag{A.1}$$

where $b_{ik}^{(\pm)}$ generalize b^{EM} to the case of the massless EW bosons. Similarly to eq. (3.5) they can be represented as the sum

$$b_{ik}^{(\pm)} = \delta_{ik}\, b_{\mathrm{QCD}}^{(\pm)} + a_{ik} + D_{ik}^{(\pm)}. \tag{A.2}$$

Term $b_{\mathrm{QCD}}^{(\pm)}$ in eq. (A.2) is defined in eq. (2.16), a_{ik} can easily be obtained from eq. (3.6), adding to a^{EM} the Z and W -boson couplings:

$$a_{uu} = a_{dd} = 4\pi\alpha Q_u^2 + g_{uZ}^2 = 4\pi \frac{\alpha}{\sin^2\theta_W} \frac{(1 + Y^2\tan^2\theta_W)}{4}\,, \qquad a_{ud} = a_{du} = \frac{g^2}{2} = \frac{4\pi\alpha}{2\sin^2\theta_W} \tag{A.3}$$

and $D_{ik}^{(\pm)}$ are generalizations of $D_{\mathrm{EM}}^{(\pm)}$ defined in eq. (3.7). It is convenient to represent $D_{ik}^{(\pm)}$ in the following way (cf eq. (3.7)):

$$\begin{aligned} D_{uu}^{(\pm)} = D_{dd}^{(\pm)} &= -\frac{4\alpha C_F}{b\sin^2\theta_W}\Bigg[\frac{(1 + Y^2\tan^2\theta_W)}{4}[1\mp 1]e^{\omega\eta}\int_{-1}^{\infty} dt e^{-\omega\eta t}\ln t \\ &\quad + \int_0^{\infty} d\rho e^{-\omega\rho}\left(\frac{(3+Y^2\tan^2\theta_W)}{4}\frac{\rho(\rho+\eta)}{(\rho+\eta)^2+\pi^2} \mp \frac{(1+Y^2\tan^2\theta_W)}{4}\frac{\rho}{\rho+\eta}\right)\Bigg] \\ &\quad - \frac{4\alpha^2}{\omega^2\sin^4\theta_W}\left[[1\mp 1]\frac{(1 + Y^2\tan^2\theta_W)^2}{16} + \frac{(-1 + Y^2\tan^2\theta_W)}{8}\right], \\ D_{ud}^{(\pm)} = D_{du}^{(\pm)} &= -\frac{2\alpha C_F}{b\sin^2\theta_W}[1\mp 1]e^{\omega\eta}\int_{-1}^{\infty} dt e^{-\omega\eta t}\ln t \pm \frac{2\alpha C_F}{b\sin^2\theta_W}\int_0^{\infty} d\rho e^{-\omega\rho}\frac{\rho}{\rho+\eta} \\ &\quad - \frac{4\alpha^2}{\omega^2\sin^4\theta_W}[1\mp 2]\left[\frac{(-1 + Y^2\tan^2\theta_W)}{8}\right]. \end{aligned} \tag{A.4}$$

When α_s is fixed, the expressions for $D_{uu}^{(\pm)}$ and $D_{dd}^{(\pm)}$ look more simple and instead of eq. (A.4)) we obtain:

$$\begin{aligned} D_{uu}^{(\pm)} = D_{dd}^{(\pm)} &= -\frac{8\alpha\alpha_s C_F}{\omega^2\sin^2\theta_W}\frac{(3 + Y^2\tan^2\theta_W)}{4}[1\mp 1] \\ &\quad - \frac{4\alpha^2}{\omega^2\sin^4\theta_W}\frac{(1 + Y^2\tan^2\theta_W)^2}{16}[1\mp 1] - \frac{4\alpha^2}{\omega^2\sin^4\theta_W}\frac{(-1 + Y^2\tan^2\theta_W)}{8}\,, \\ D_{ud}^{(\pm)} = D_{du}^{(\pm)} &= -\frac{2\alpha\alpha_s C_F}{\omega^2\sin^2\theta_W}[1\mp 1] - \frac{4\alpha^2}{\omega^2\sin^4\theta_W}[1\mp 2]\left[\frac{(-1 + Y^2\tan^2\theta_W)}{8}\right]. \end{aligned} \tag{A.5}$$

Let us comment on eqs. (A.4), (A.5). The terms $\sim 1/b$ in eq. (A.4) (corresponding to the term $\sim \alpha\alpha_s$ in eq. (A.5) where α_s is fixed) come from the interference of the QCD and EW DL contributions. The next term in each of eqs. (A.4), (A.5) accumulate the DL contributions of virtual soft neutral EW bosons: photons and Z-bosons. A part of those terms in eq. (A.4) (and all of them in eq. (A.5)) is proportional to the signature factor $[1 \mp 1]$ and therefore vanish when the signature is positive. In other words, the non-ladder DL contributions to the amplitudes with the positive signature cancel each other totally when couplings are fixed[5] but such a cancelation is not total when all the couplings or some of them are running. The presence of the last term in eqs. (A.4), (A.5) demonstrates explicitly that accounting for the soft W-boson exchanges breaks such a cancelation for $D^{(+)}_{ik}$ even in the case when the couplings are fixed. However, when α_s is kept fixed, the total summation over flavors for $D^{(+)}_{ik}$ of eqs. (A.5) leads to the zero contribution of the non-ladder graphs:

$$D^{(+)}_{uu} + D^{(+)}_{ud} + D^{(+)}_{dd} + D^{(+)}_{du} = 0\ . \tag{A.6}$$

Eq. (A.6) is quite similar to the QCD result for $D^{(+)}_{\mathrm{QCD}}$ with fixed α_s obtained first in ref. [19] because summation over flavors in eq. (A.5) is equivalent to summation over colors in QCD. As b_{ik} are now fixed, we can solve eqs. (A.1). Combining eqs. (A.2), (A.3), (A.4) we see that $b_{uu} = b_{dd}$, $b_{ud} = b_{du}$ and therefore eq. (A.1) reads that $H_{uu} = H_{dd}$ and $H_{ud} = H_{du}$. After that eq. (A.1) can easily be solved:

$$\begin{aligned} H_{uu} &= H_{dd} = \frac{1}{2}\Big[\omega - E\Big]\ , \\ H_{ud} &= H_{du} = \frac{\widetilde{b}_{ud}}{E} \end{aligned} \tag{A.7}$$

where

$$\widetilde{b}_{uu} = \frac{b_{uu}}{8\pi^2}\ , \qquad \widetilde{b}_{ud} = \frac{b_{ud}}{8\pi^2}\ , \qquad E = \sqrt{\frac{\omega^2 - 4\widetilde{b}_{uu} + \sqrt{(\omega^2 - 4\widetilde{b}_{uu})^2 - 16\widetilde{b}^2_{ud}}}{2}}\ . \tag{A.8}$$

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